THEORY OF IDEAL ELECTRIC ARC IN COAXIAL PLASMATRON WITH AXIAL MAGNETIC FIELD IN THE PRESENCE OF GAS FLOW

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The qualitative pattern of the interaction of an electric arc with an axial magnetic field and the working medium in a coaxial plasmatron (Fig. 1) is simpler than in a plasmatron with vortex stabilization of the arc because of the fact that for the limited arc length in the coaxial plasmatron there is no need to resort to the breakdown (shunting) phenomenon between the arc and the wall. However, an analytic solution for the arc form and its rate of rotation is known only for the case of no longitudinal gas flow [1].

The imposition of axial gas flow complicates the problem from the viewpoint that, generally speaking, the mechanism which sets into motion along the axis the arc elements which arise on the inner electrode of the plasmatron and travel toward the outer electrode is not entirely obvious if the effects near the electrodes are taken into account. Moreover, the question arises of determining the angular velocity of the arc as a whole, which is closely related with the initial conditions for the existence of the arc elements on the inner electrode (the choice of the value of this rate in [1] is not justified by any physical considerations). In the present case this problem is still further complicated by the appearance of an additional source of influence — the axial gas flow.

An analysis is presented of an idealized scheme of electric-arc behavior under these conditions. The idealization lies in the fact that the effects in the electrode region and the shunting phenomenon between the arc and the electrodes are not taken into account, and the rate of rotation of the arc as a whole is assumed constant in time. Moreover, several assumptions are introduced which are specified below. The objective of the study is to clarify at least the qualitative nature of the influence of such parameters as the current strength, magnetic field, gas flow velocity, and geometric dimensions of the plasmatron, primarily on the arc voltage.

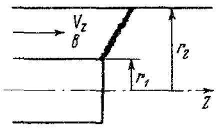

Fig. 1

1. A schematic of the coaxial plasmatron is shown in Fig. 1. The electric arc burns between cylindrical coaxial electrodes with radii r_1 and r_2. The arc is caused to rotate under the influence of the magnetic field of strength B, which is uniform radially and lengthwise. The axial flow with velocity V_z, uniform across the section, blows the arc along the plasmatron axis.

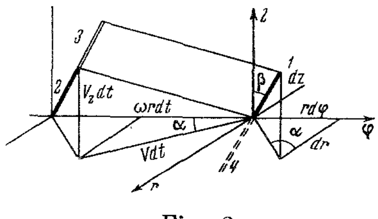

Fig. 2

Let us examine in more detail the pattern of arc-element motion under the action of the magnetic field and gas flow (Fig. 2).

At some initial time the arc element of length dl occupies the position 1 at the radius r from the plasmatron axis. Under the influence of the interaction force between the arc current and the external magnetic field the arc element displaces in the r, φ plane (where φ is the angle measured from the point of contact of the arc with the surface of the inner electrode at the time in question) perpendicular to the plane formed by the arc element and the z-axis, which emanates from the end of the element. As the motion continues under the influence of the electromagnetic force the arc element also displaces along the z-axis under the action of the gas flow. We assume that the velocity of the arc element along the z-axis equals the flow velocity V_z. This assumption is based on the fact that the arc is not separated from the gas flow by any sort of rigid boundary and the velocity of the directed motion of the arc particles under the influence of the aerodynamic forces can not differ from the velocity of nearby particles of the surrounding medium, which is also confirmed by observations of the motion of radial segments of the arc in plasmatrons with vortex stabilization [2].

After the time interval dt the arc element in question occupies position 2. (Here, however, we neglect the interaction force on the arc segment from the arc self-magnetic field, and also the flow swirl under the influence of rotation of the arc column.) The place of element 1 at this same radius r is occupied by element 3, which has arrived from position 4 which it occupied at the initial time. Instruments which follow the rotation of the arc as a whole record the distance between elements 1 and 3 along the circle of radius r as $\omega r dt$, where ω is the angular velocity of the arc. In this scheme the time interval dt is not arbitrary, since element 1 in position 3 is an extension of element 2 of the same length dl as that of element 1. This picture clearly illustrates that the angular velocity of the arc sensed by the instruments does not coincide with the velocity of the individual arc elements, in spite of the fact that they are connected by the continuous time-stationary curve describing the form of the arc. The possible arc-stabilization mechanism (in the sense of limited arc burning region) in the coaxial plasmatron becomes clear from this analysis. Under the action of the magnetic field the arc elements which arise on the inner electrode displace along the direction toward the outer electrode and disappear on the latter. This process leads to the situation in which the surface (more precisely, a region of thickness of the order of the arc transverse dimension) on which the arc elements can be located intersects the surfaces of the inner and outer electrodes, thus "stabilizing" the arc rotation zone.

The velocity V of the arc element under the action of the electromagnetic force is found, as suggested in [3] and [1], from equilibrium of this force and the aerodynamic resistance force of the element if it is examined as a rigid conductor with the effective gasdynamic transverse dimension D,

$$IB \sin \beta = {}^1/_2 c_f \rho V^2 D. \tag{1.1}$$

Here I is the arc current strength, β is the angle between the directions of the z-axis and the arc element, ρ is the approaching gas density, cf is the resistance coefficient of the arc element.

Considering that the projections of the arc element of length dl onto the axis of the cylindrical coordinate system are dr, $rd\varphi$, and dz, we find

$$\sin \beta = \left(\frac{1 + (rd\varphi / dr)^2}{1 + (rd\varphi / dr)^2 + (dz / dr)^2}\right)^{1/2}. \tag{1.2}$$

From the equality of the angles α, shown in Fig. 1, we obtain

$$\omega r / V = \sqrt{1 + (rd\varphi / dr)^2}. \tag{1.3}$$

The time interval dt is such that length of element 2 is equal to the length of element 1; therefore we have from Fig. 1 that

$$dz = V_z dt. \tag{1.4}$$

Geometric relations lead to

$$\frac{1}{V} \frac{dr}{dt} = \sin \alpha = \left(r \frac{d\varphi}{dr}\right) [1 + (rd\varphi / dr)^2]^{-1/2}. \tag{1.5}$$

Excluding dt from (1.4) and (1.5), we obtain

$$\frac{V_z}{V} = \frac{r}{\sqrt{1 + (rd\varphi / dr)^2}} \frac{d\varphi}{dr} \frac{dz}{dr}. \tag{1.6}$$

We denote by V_0 the velocity of the arc elements in the absence of longitudinal gas flow, under the assumption that the other external parameters and cf remain unchanged. Then from (1.1)

$$V_0^2 = \frac{2IB}{c_f \rho D_0}. \tag{1.7}$$

Here we have used the quantity D_0 since, generally speaking, the transverse arc dimension must depend on the velocity of the element relative to the gas. Experiments are described in [3] which made it possible to show that the average current density j of the transversely blown arc is proportional to the gas velocity relative to the arc. Since $j \sim I/D^2$, we can obtain

$$D / D_0 = \sqrt{V_0/V}. \tag{1.8}$$

Comparing (1.1), (1.7), and (1.8) we find

$$V / V_0 = (\sin\beta)^{2/3}. \tag{1.9}$$

We introduce the notation

$$\xi = \left(r\frac{d\varphi}{dr}\right)^2, \quad \zeta = \left(\frac{dz}{dr}\right)^2, \quad \Omega = \left(\frac{\omega r}{V_0}\right)^2, \quad \eta = \left(\frac{V_z}{V_0}\right)^2.$$

After transformations we obtain the system of equations describing the shape of the arc,

$$1 + \frac{\eta}{\Omega}\frac{1+\xi}{\xi} = \left(\frac{1+\xi}{\Omega}\right)^{3/2}, \qquad \zeta = \frac{\eta}{\Omega}\frac{(1+\xi)^2}{\xi}. \tag{1.10}$$

The voltage on the arc is

$$U = \int_{r_1}^{r_2} E\sqrt{1+\xi+\zeta}\,d\cdot \tag{1.11}$$

Here E is the electric field intensity in the arc. It was shown in [3] that the minimum voltage principle can be used to obtain the dependence of the voltage of the transversely blown arc on the blowing velocity and current strength, which has been confirmed experimentally:

$$E = c\,(V^2 / I)^{1/3}. \tag{1.12}$$

Using (1.12) and (1.10), Eq. (1.11) can be written in the form

$$U = c\left(\frac{V_0^2}{I}\right)^{1/3} \frac{r_1}{2\sqrt{\Omega_1}} \int_{\Omega_1}^{\Omega_2} \left(\frac{1+\xi}{\Omega}\right)^{11/12} d\Omega. \tag{1.13}$$

Here Ω_1 and Ω_2 correspond to the values of Ω at the radii r_1 and r_2. For $V_z = 0$ it follows from the first equation that

$$\Omega = 1 + \xi.$$

In this case the voltage is defined by the relation

$$U_0 = c\left(\frac{V_0^2}{I}\right)^{1/3} \frac{r_1}{2\sqrt{\Omega_{10}}} (\Omega_{20} - \Omega_{10}). \tag{1.14}$$

Since the angular velocity ω of the arc can depend on the blowing velocity, the values of Ω_1 and Ω_2 in the presence of gas flow may differ from these same quantities Ω_{10} and Ω_{20} in the absence of gas flow.

It is interesting to reduce the equation for the arc voltage to dimensionless form, using the well-known [4,5] similarity criteria for arcs: the voltage criterion φ, the current criterion ("energetic criterion") i, and the Reynolds number R:

$$\varphi = \frac{U r_1 \sigma}{I}, \quad i = \frac{I}{\sqrt{r_1^3 \sigma h \rho V_z}}, \quad R = \frac{\rho V_z r_1}{\mu}.$$

Here σ is the characteristic electrical conductivity of the arc; ρ, h, and μ are the characteristic values of the density, enthalpy, and viscosity of the approaching flow.

It is natural to take as the characteristic linear dimension of the problem the quantity r_1, which follows from (1.14). We can find

$$\varphi = \frac{\beta}{2\sqrt{\Omega_1} i^{4/3} \eta^{1/3}} \int_{\Omega_1}^{\Omega_2} \left(\frac{1+\xi}{\Omega} \right)^{11/12} d\Omega \,. \tag{1.15}$$

where β is a constant, which according to similarity theory is independent of the properties of the gaseous medium and the arc parameters

$$\beta = c\sigma^{1/3} (h\rho)^{-2/3}.$$

Hence, in addition, it follows that the electric field intensity

$$E = \beta \left(\frac{h^2\rho^2 V^2}{\sigma I} \right)^{1/3}.$$

It is typical that (1.15) shows that the voltage criterion φ is independent of the Reynolds number.

2. We write the first equation (1.10) in the form

$$\Omega = \frac{1}{x} + \frac{\eta}{x^{3/2} - 1}, \quad x = \frac{1+\xi}{\Omega}. \tag{2.1}$$

Using (2.1), we can integrate (1.15). We then solve the problem approximately, neglecting the difference between the exponent 11/12 in (1.16) and 1. This makes it possible to obtain a simple analytic expression for φ. Integrating (initially by parts), we find

$$\varphi = \frac{2\beta}{\sqrt{\Omega_1} i^{4/3} \eta^{1/3}} [F(x_2) - F(x_1)]. \tag{2.2}$$

Here

$$F(x) = -\ln x + \eta \left[\frac{x}{x^{3/2} - 1} + \ln (1 + \sqrt{x} + x) - \frac{2}{3} \ln (x^{3/2} - 1) - \frac{2}{3} \operatorname{arc\,tg} \frac{2\sqrt{x} + 1}{\sqrt{3}} \right]. \tag{2.3}$$

Without longitudinal gas flow (according to [1] $\Omega_{10} = 1$) the expression for φ has the form

$$\varphi_0 = \frac{2\beta}{i_*^{4/3}} (d^2 - 1) \qquad \left(d = \frac{r_2}{r_1} \right), \tag{2.4}$$

where i_* is the current criterion, defined in terms of the velocity V_0.

Since even in the presence of the longitudinal gas flow the product $i^{4/3} \eta^{1/3}$ is independent of V_z, we find the ratio

$$\frac{\varphi}{\varphi_0} = \frac{F(x_2) - F(x_1)}{\sqrt{\Omega_1} (d^2 - 1)}, \tag{2.5}$$

which expresses the influence of the longitudinal gas flow velocity across the arc. To close the resulting system of equations we must determine the magnitude of the angular velocity of the arc ω (or Ω_1), since the equations presented above admit an arbitrary value of ω. In [1], in the absence of longitudinal gas flow it was assumed that $\Omega_1 = 1$. The following arguments may be used to justify this assumption:

a) the condition $\Omega = 1$ satisfies the Shteenbeck minimum principle [6] in the formulation: for given external conditions the minimal possible voltage drop is established across the arc;

b) as the arc elements are formed on the inner electrode they must obviously be positioned normal to the electrode surface. Otherwise the arc elements would separate from the electrode at the instant of formation. This hypothesis also yields as a result $\Omega_1 = 1$.

In the presence of longitudinal gas flow the application of the Shteenbeck minimum principle in this formulation leads to a physically invalid result. In fact, in this case it is found that Ω_1 (or the initial arc inclinations ξ_1 and ζ_1, which are directly related with Ω_1) depends on d. However, it is difficult to imagine that a change of only the diameter of the outer electrode while retaining all the other parameters unchanged can lead to change of the arc element motion pattern near the inner electrode. In fact, the information on the arc behavior flows in the direction of displacement of the arc elements, i.e., from the inner electrode toward the outer. In the adopted analytic scheme there is obviously no feedback mechanism, i.e., the behavior of arc elements on the outer electrode can not affect arc burning conditions at the inner electrode.

The direct application of another hypothesis to the given problem in the formulation indicated above, i.e., under the assumption of uniform axial gas flow velocity profile, also leads to a result which is not observed physically (it follows from (1.10) that $\Omega_1 = 0$). However, if we consider the actual gas flow pattern this hypothesis may yield a more or less reasonable result. In fact, the velocity profile in the interelectrode gap is nonuniform. Within the limits of the boundary layer near the electrode walls the velocity changes from the freestream velocity to zero at the electrode walls. Thus, the conditions used in [1] are realized in the immediate vicinity of the wall of the inner electrode, which makes it possible to assume that the arc elements in the vicinity of the electrode are arranged normal to its surface. Then it is natural to assume that $\Omega_1 = 1$ on the electrode surface. If the boundary layer dimension is small in comparison with the interelectrode gap, then at the beginning of the uniform velocity profile region the quantity Ω (for the given problem this is Ω_1) must differ very little from unity. The results of these obviously inexact arguments may be formulated in the form of a hypothesis: the angular velocity of the arc as a whole is independent of the longitudinal gas flow velocity. The condition $\Omega_1 = 1$ closes the resulting system of equations.

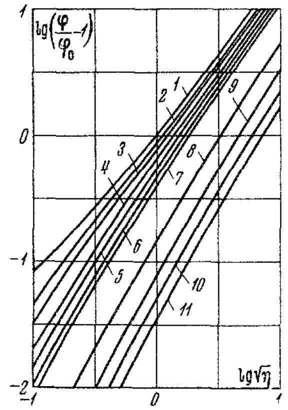

Fig. 3

The results of calculations of the ratio φ/φ_0 are shown in Fig. 3: the numerals 1–11 correspond to the values d = 1.0, 1.1 1.2, 1.4, 1.8, 2.0, 4.0, 6.0, 8.0, 10.0.

With accuracy adequate for calculations these data are approximated by the expression

$$\frac{\varphi}{\varphi_0} = 1 + 10^{-3.520\,(d^{0.15}-1)}\eta^{0.635d^{0.185}}. \tag{2.6}$$

3. Some qualitative properties of the arc in the scheme being considered can be obtained from examination of the limiting relations which follow from the formulas derived.

Thus, it can be shown that the current strength dependence of the voltage over a wide range of variation of the current strength cannot be described by a single (as is usually done) power-law dependence with a constant exponent on I.

Since (according to [3]) $V_0 \sim I^{1/3}$, increase of the current strength means a decrease of η,

$$\frac{\varphi}{\varphi_0} \approx \eta^{2/3}\frac{d^{2/3}-1}{d^2-1} \quad (\eta \to \propto), \qquad \frac{\varphi}{\varphi_0} \approx 1 - \frac{1}{3}\frac{\eta \ln \eta}{d^2-1} \to 1 \quad (\eta \to 0) \tag{3.1}$$

Bearing in mind the above-mentioned dependence of V_0 on I and taking account of (2.4), we can find that for small and large values of the current strength the voltage will be, respectively,

$$U \sim I^{-5/9}, \qquad U \sim I^{-1/3}.$$

Thus, if we examine a wide range of variation of the current strength with the other parameters unchanged the volt-ampere curve may alter its appearance considerably. We also see that the volt-ampere curve of the plasmatron within the assumptions adopted has a negative slope, although with increase of the current strength the derivative

$\partial U/\partial I$ increases. It appears to be possible to obtain a volt-ampere curve with positive slope in the coaxial plasmatron. If we accept the validity of the suggestion of Bron [3] that the voltage of the transversely blown arc can be represented in the form of a linear superposition of expression (1.12) and the voltage of the freely burning arc, then for large values of the current strength, when the voltage of the freely burning arc, whose volt-ampere curve has a U-shaped form, increases with increase of the current, the influence of this contribution can become significant and lead to increase of the over-all voltage across the arc with increase of the current.

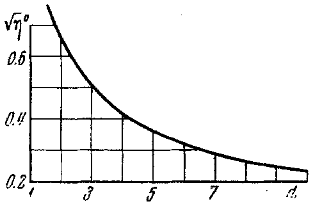

Fig. 4

The influence of a magnetic field on the voltage across the arc is ambiguous. From the limiting equations (3.1), if we adopt the relation $V_0 \sim \sqrt{B}$ (following [3]), we can find that for large values η (high flow velocities, small magnetic fields, small currents)

$$U \sim B^{-1/3};$$

for small values of η (low flow velocities, high magnetic fields, large currents)

$$U \sim B^{+1/3}.$$

Such dependence of the voltage on the magnetic field is quite understandable. For zero magnetic field the coaxial plasmatron degenerates into a plasmatron with longitudinal blowing of the arc, and in the present scheme the voltage across the arc must be infinite, since the arc length is infinite. Increase of the magnetic field leads to shortening of the arc length as a result of the appearance of the mechanism for transport of the arc elements from the inner electrode in the direction toward the outer electrode. For sufficiently high fields increase of the magnetic field has very little influence on the arc length and increase of the field results primarily in increase of the intensity of the electric field. Equation (2.5) makes it possible to calculate simply the values $\eta = \eta^\circ$ for which there is reversal of the direction of the influence of the magnetic field on the voltage across the arc. Figure 4 shows the quantity $\sqrt{\eta}^\circ$ versus d. Just as in the case of the volt-ampere curve, we see that over a wide range of variation of the magnetic field intensity the influence of the magnetic field on the voltage drop across the arc cannot be described by a single power-law relation with a constant exponent on B.

The approximation (2.6) and expression (2.4) show that the dependence of the voltage across the arc on the geometric parameter d is described by the expression

$$\varphi \sim d^2 - 1,$$

where in the range of variation of d examined the corrections to this law are small.

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